**a. Build 99% Confidence Interval Using Sample Standard Deviation**

Assuming the sample is representative of the population, construct a 99% confidence interval for the mean number of characters printed before the print-head fails using the sample standard deviation. Explain the steps you take and the rationale behind using the t-distribution for this task.

Ans:

**Data (is given in the question)**

*A total of 15 print-heads were randomly selected and tested until failure. The durability of each print-head (in millions of characters) was recorded as follows:*

*1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29*

Since, sample of 15 print-heads given so let’s calculate sample mean and sample standard deviation

Sample mean (x̄) = (1.13 + 1.55+ 1.43 + 0.92+………+1.29)/15 = 18.58/15

= 1.24 millions of characters

Sample standard deviation (s):

Sample variance = [(1.13-1.24)2+ (1.55-1.24)2 +………+(1.29-1.24)2] /15

= 0.5224/15 = 0.03482

Sample standard deviation (s) = sqrt(0.03482) = 0.187 millions of characters

In (a): we have to construct a 99% confidence interval for the mean number of characters printed before the print-head fails using sample standard deviation (s) which means that we have to estimate population mean using sample standard deviation at 99% confidence interval.

To construct the confidence interval, we can use t-distribution for 2 main reasons:

Reason 1: When sample size is small (less than 30)

Here, sample size (n) = 15 which is less than 30

Reason 2: When population standard deviation is unknown

Here, we knows only sample standard deviation

Since, we met both these 2 conditions, therefore, we can use t-distribution to construct 99% confidence interval

Let’s calculate t-score:

Step1: confidence interval = 99% => confidence level (c) = 0.99

Step2: significance level (α) = 1-c = 1-0.99 = 0.01

Step3: t α/2 = t0.01/2 = t 0.005 with degree of freedom = n-1 = 15-1=14

From t- distribution table : t 0.005 with (degree of freedom=14) = 2.97684

Which means 2.97684 is the t-value we need for our 99% confidence interval to estimate the population mean

**b. Build 99% Confidence Interval Using Known Population Standard Deviation**

If it were known that the population standard deviation is 0.2 million characters, construct a 99% confidence interval for the mean number of characters printed before failure.

Ans: Population standard deviation (σ) = 0.2 million characters

We have to construct the 99% confidence interval for the mean number of characters printed before failure using Population standard deviation which means we have to estimate the population mean using population standard deviation at 99% confidence interval.

Since σ is given which means population standard deviation is known to us, therefore, we can use Z-distribution to estimate the population mean at 99% confidence interval.

Mathematically,

z = (x̄ – μ) / (σ / √n) ………….equation 1

where, x̄ : sample mean

μ: population mean

σ : population standard deviation

n: sample size

x̄= 1.24 millions of characters (already calculated above)

σ = 0.2 million characters (given)

n= 15

μ =?

Z= 2.58 (calculated below)

Z- score at 99% confidence interval:

CL (Confidence Level) = 0.99, A= (1+CL)/2

A (Area under Normal distribution curve)= (1+0.99)/2 = 0.995

Search 0.995 in Normal distribution Table for the corresponding row & column, where we can see 0.995 is very close to 0.9949 & 0.9951

From this table, Z-score of 0.9949 = 2.57

Z-score of 0.9951 = 2.58

Therefore, Z-score of 0.995 = mean of these values = (2.57+2.58)/2 = 2.575

Hence, Z-score at 99% Confidence Interval = 2.58

Put all values in equation 1, we get μ = 1.107 millions of characters

**It means 1.107 is the population mean at 99% confidence interval.**